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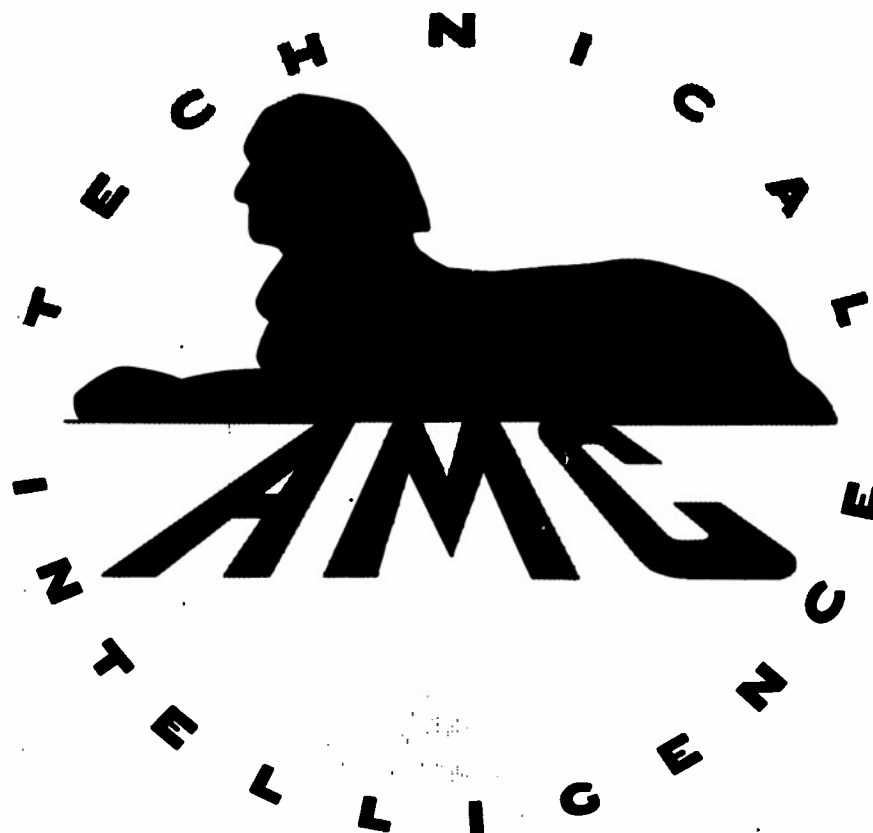
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Dickson, R.

Aerodynamics (2)

20354

Wings and Airfoils (6)

Airfoils - Aerodynamics (07710);

Aero-1790

Airfoils - Swept-back - Drag (08280)

Comparison of two methods of calculating aerodynamic loading on an aerofoil with large sweepback and small aspect ratio

Royal Aircraft Establishment, Farnborough, Hants

GtU.Brit. Eng.

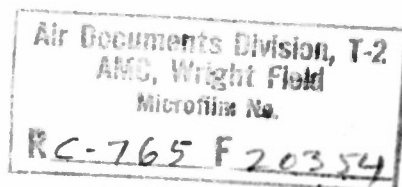
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tables, graphs, dwrgs

The aerodynamic characteristics of an airfoil, of large sweep-back and small aspect ratio at Mach of 0.9, were calculated by means of Falkner's nine-point solution and also by a simplified version of Falkner's solution. The two methods gave comparative results except for the aerodynamic-center calculation which, by the simplified method, was 0.05-chord in front of that of the original method. This results from the omission, in the simplified method, of the variation in chord-wise position of the local aerodynamic center.



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# ROYAL AIRCRAFT ESTABLISHMENT

## Farnborough, Hants.

### COMPARISON OF TWO METHODS OF CALCULATING AERODYNAMIC LOADING ON AN AEROFOIL WITH LARGE SWEEPBACK AND SMALL ASPECT RATIO

by

R. DICKSON, B.A.

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June, 1946

ROYAL AIRCRAFT ESTABLISHMENT, FARNBOROUGH

Comparison of Two Methods of Calculating  
Aerodynamic Loading on an Aerofoil with  
Large Sweepback and Small Aspect Ratio

by

R. Dickson, B.A.

R.A.E. Ref: Aero.S/1733/R/171

SUMMARY

R The purpose of the present note is to compare the aerodynamic characteristics of an aerofoil of large sweepback and small aspect ratio, calculated by two methods:-

(i) Falkner's nine-point solution

and (ii) a simplified version of this method using only a lifting line on the quarter chord and three pivotal points, situated on the three-quarter chord line.

The results are used to give information on the characteristics of an aerofoil with straight taper and moderate aspect ratio and sweepback in compressible flow at a Mach number 0.9.

The results show that the spanwise loading curves, lift curve slope, induced drag, local lift coefficient at a given  $C_L$ , and bending centre on half the wing, estimated by the two methods, are comparable, but the aerodynamic centre calculated by (ii) is about 0.058 in front of that calculated by (i). The main reason for this appears to be the omission in method (ii) of the variation in chordwise position of the local aerodynamic centre.

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## 1 Introduction

An attempt is being made to develop a reasonably quick method of calculating the aerodynamic loading of swept-back wings. This is required for the estimation of stability derivatives, both longitudinal and lateral and the effects of wing distortion on stability, as well as for stressing purposes.

Falkner<sup>1</sup> considers a lifting plane composed of horse-shoe vortices approximating to a continuous loading. The induced downwash is calculated at a number of pivotal points and equated to the slope of the aerofoil. For symmetrical loading, 9 points are used on the half wing, 3 points chordwise at each of 3 spanwise positions. In its simplest form<sup>2</sup> the horse-shoe vortices are along the  $\frac{1}{2}$  chord line only, and the downwash is calculated at 3 pivotal points on the  $\frac{2}{3}$  chord line. This simplified method is in principle similar to the methods of Lütterperl<sup>3</sup> and Weissinger<sup>4</sup>.

Falkner has shown<sup>5</sup> that the wing loading due to incidence obtained by the simplified method on a wing with 28° sweepback and 5.8 aspect ratio is a good approximation to the 9 point solution. The work described here was undertaken to compare the two methods in an extreme case of small aspect ratio, 2, and large sweepback, approximately 60°.

It is realized however, that there is some doubt as to whether any vortex sheet method is sufficiently reliable for calculating the aerodynamic characteristics of aerofoils of very small aspect ratio.

The aerofoil used for the calculation has a taper ratio of 3.25:1, aspect ratio 2, and sweepback 59° 8', and is considered to be a flat plate in this calculation. This wing is called Aerofoil I. A brief outline of the procedure adopted for the calculation is given.

The results are also applied to the estimation of the aerodynamic loading on Aerofoil II (Fig. 3) in compressible flow at  $M = 0.9$ . This aerofoil has an aspect ratio 4.64, taper ratio 3.25:1, and sweepback 36° 5'.

## 2 Calculation of Aerodynamic Characteristics, using Falkner's Nine-Point Method

### 2.0 General

The method is given in Reference 1. The vorticity  $k$  is represented by the formula

$$\begin{aligned} \frac{ko}{8sV \tan \alpha} = & \sqrt{(1-\eta^2)} \cdot (a_0 + c_0 \eta^2 + e_0 \eta^4) \cot \frac{\theta}{2} \\ & + \sqrt{(1-\eta^2)} \cdot (a_1 + c_1 \eta^2 + e_1 \eta^4) \sin \theta \\ & + \sqrt{(1-\eta^2)} \cdot (a_2 + c_2 \eta^2 + e_2 \eta^4) \sin 2\theta \dots\dots\dots(1) \end{aligned}$$

$$\text{or } \frac{ko}{8sV \tan \alpha} = F_0(\eta) \cot \frac{\theta}{2} + F_1(\eta) \sin \theta + F_2(\eta) \sin 2\theta, \dots\dots(2)$$

where  $\eta$ ,  $\theta$  are defined so that, if  $y$  is the distance of any point of the aerofoil from the centre line,

$$y = s\eta \dots\dots\dots(3)$$

and  $\cos \theta = \frac{2x}{c}$ , .....(4)

where  $x$  is the distance of any point of the aerofoil from the mid-point of the chord through it.

The total circulation  $K$  at any section  $\eta$  is given by

$$\frac{K}{4s V \tan \alpha} = \pi \left[ F_0(\eta) + F_1(\eta)/2 \right] = A_1 \sin \phi + A_3 \sin 3\phi + A_5 \sin 5\phi, \dots (5)$$

where  $A_1 = \frac{\pi}{16} [16a_0 + 8a_1 + 4c_0 + 2c_1 + 2e_0 + e_1]$ , ..(6)

$$A_3 = \frac{\pi}{32} [8c_0 + 4c_1 + 6e_0 + 3e_1], \dots (7)$$

$$A_5 = \frac{\pi}{32} [2e_0 + e_1], \dots (8)$$

and  $\eta = \cos \phi$ . .....(9)

The main work is the calculation of the coefficients  $a_0, a_1, a_2, c_0, c_1, c_2, e_0, e_1, e_2$ , which are chosen to satisfy the condition that the downwash at each of nine pivotal points of the wing is equal to the slope  $\tan \alpha$  of the aerofoil. An outline of the procedure is given below in para. 21. When this has been done, the values of  $C_L, C_{D1}$ , bending centre\*, and local lift coefficient are obtained from the following formulae:-

$$\frac{dC_L}{d\alpha} = \frac{4\pi s^2 A_1}{s}, \dots (10)$$

$$C_{D1} = \frac{C_L^2}{\pi A} \cdot \frac{A_1^2 + 3A_3^2 + 5A_5^2}{A_1^2}, \dots (11)$$

The distance of the bending centre from the centre line

$$= \frac{s}{\pi A_1} [1.3333 A_1 + 0.8000 A_3 - 0.1905 A_5], \dots (12)$$

The local lift coefficient  $C_{LL}$  is given by

$$\frac{C_{LL}}{C_L} = \frac{8\pi \cdot (F_0(\eta) + \frac{1}{2}F_1(\eta))}{\frac{c}{s} \cdot \frac{dC_L}{d\alpha}}, \dots (13)$$

The local aerodynamic centre at any spanwise position is at a distance  $\frac{1}{4c} \left[ \frac{F_0(\eta) + F_1(\eta) - \frac{1}{2}F_2(\eta)}{F_0(\eta) + \frac{1}{2}F_1(\eta)} \right]$  behind the local leading edge.

\*Bending centre is the spanwise position of the centre of load on half the aerofoil due to a change of incidence.

By a Simpson integration of the moment of the load about the datum line (shown in Fig. 1) we obtain the aerodynamic centre for the whole wing. Its position can then be stated relative to the mean chord.

## 2.1 Procedure for calculating the coefficients

The continuous vorticity is replaced by a system of horse-shoe vortices of width  $1/40$  of the span, and the downwash produced by these horse-shoe vortices at pivotal points on the wing calculated. The layout of the vortices and the positions of the pivotal points are shown in Fig. 2. The vortices at  $\eta = 0.9625$  are corrector vortices, giving the effect of the tip, and are only one quarter the width of the other horse-shoe vortices.

Chordwise, the 4 vortices are at 0.125, 0.375, 0.625, and 0.875 of the local chord from the leading edge, and their values are determined by the conditions that their sum is equal to the chordwise integral of the continuous loading at this spanwise position, and that the downwash produced by the 4 vortices at 0.25, 0.5 and 0.75 of the local chord from the leading edge in 2 dimensional flow is the same as that produced by the continuous chordwise vorticity. When these conditions are satisfied, the values  $k_1, k_2, k_3, k_4$ , of the 4 chordwise vortices are given by:-

$$k_1 = 0.2734 \pi c K_1 + 0.0488 \pi c K_2 + 0.0732 \pi c K_3 \dots\dots\dots(14)$$

$$k_2 = 0.1172 \pi c K_1 + 0.0762 \pi c K_2 + 0.0481 \pi c K_3 \dots\dots\dots(15)$$

$$k_3 = 0.0705 \pi c K_1 + 0.0762 \pi c K_2 - 0.0381 \pi c K_3 \dots\dots\dots(16)$$

$$k_4 = 0.0391 \pi c K_1 + 0.0488 \pi c K_2 - 0.0732 \pi c K_3 \dots\dots\dots(17)$$

where

$$k = K_1 \cot \frac{\theta}{2} + K_2 \sin \theta + K_3 \sin 2\theta \dots\dots\dots(18)$$

is the vorticity at the spanwise position which is being considered.

The following is then the procedure. First, a table is constructed of distances of the leading edge from the datum line, and of the magnitude of the local chord, in terms of the span, for the values of  $\eta$  from 0 to 1 by intervals of 0.05. Then, the distances chordwise from the datum line of the horse-shoe vortices are tabulated in terms of the width of a vortex, i.e. one fortieth of the span. The spanwise and chordwise distances of each of the 84 horse-shoe vortices from each of the nine pivotal points are then deduced, and tabulated. From tables of the downwash function  $F$ , defined in Ref. 1, the values of  $F$  are calculated at each pivotal point for the downwash created by each horse-shoe vortex.

For the calculation of the downwash at a pivotal point created by vortices at a big distance spanwise, i.e. when the distance spanwise is more than 4 times the vortex width, instead of using all chordwise vortices, it is sufficient to concentrate the term in  $\cot \frac{\theta}{2}$  at the centre of pressure of a load proportional to  $\cot \frac{\theta}{2}$ , i.e. at the quarter chord, and similarly the term in  $\sin \theta$  at the centre of pressure of a load proportional to  $\sin \theta$ , i.e. the mid-chord, while the term in  $\sin 2\theta$ , giving a total circulation of zero, is omitted.

The downwash at each pivotal point is thus obtained by summation, and this is equated to the slope of the normal at the pivotal point. This gives nine equations for the unknowns,  $a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8$ , which are then easily calculated.

The rest of the calculation is described in para. 2.0, and only amounts to substitution in known formulae.

### 3 The Simplified Method

In this method, used in Ref. 2, the vorticity is assumed to be concentrated at the quarter chord. Formula (1) is reduced to one term only, viz:-

$$\frac{kc}{8s V \tan \alpha} = (a_0' + a_0' \eta^2 + e_0' \eta^4) \sqrt{(1-\eta^2)} \cot \frac{\theta}{2} = G_0(\eta) \cot \frac{\theta}{2} \quad (19)$$

To calculate the coefficients  $a_0'$ ,  $a_0'$ ,  $e_0'$ , 3 pivotal points only are needed. They are chosen on the three-quarter chord, namely those marked 3, 6, and 9 on Fig. 2. The procedure is similar to that described in para. 2.1, but the labour is much reduced.

The formulae to be used to calculate the aerodynamic characteristics are:-

$$\frac{dC_L}{d\alpha} = \frac{4\pi s^2}{s} A_1' \quad (20)$$

$$C_{Di} = \frac{C_L^2}{\pi A} \cdot \frac{A_1'^2 + 3 A_3'^2 + 5 A_5'^2}{A_1'^2} \quad (21)$$

where the total circulation  $K$  at any section  $\eta$  is given by

$$\frac{K}{4s V \tan \alpha} = \pi G_0(\eta) = A_1' \sin \phi + A_3' \sin 3\phi + A_5' \sin 5\phi \quad (22)$$

$$\text{and} \quad A_1' = \frac{\pi}{8} [8a_0' + 2e_0' + e_0'] \quad (23)$$

$$A_3' = \frac{\pi}{16} [4a_0' + 3e_0'] \quad (24)$$

$$A_5' = \frac{\pi}{16} \cdot e_0' \quad (25)$$

The distance of the bending centre from the centre line

$$= \frac{s}{\pi A_1'} [1.3333 A_1' + 0.8000 A_3' - 0.1905 A_5'] \quad (26)$$

The local lift coefficient  $C_{LL}$  is now given by

$$\frac{C_{LL}}{C_L} = \frac{8\pi \cdot G_0(\eta)}{\frac{s}{s} \cdot \frac{dC_L}{d\alpha}} \quad (27)$$



Since it is assumed that the load is concentrated at the quarter-chord line, the aerodynamic centre of the aerofoil is readily obtained from the bending centre for aerofoils with a straight quarter-chord line, and by a Simpson integration of the moment of the load about the datum, divided by the total load, for an aerofoil with a quarter chord line of any form.

#### 4 Results: Comparison of the two methods for Aerofoil I

Tables I and II give the values of the coefficients  $a_0, a_1, a_2, c_{00}, c_{01}, c_{02}, c_0, c_1, c_2$ , obtained by the nine-point method, and  $a_0', c_{00}', c_0'$  obtained by the simplified method respectively.

Using the formulae (6) to (13) and (20) to (27) we obtain the following comparison:-

Aerodynamic characteristic	9 point method	Simplified method
$\frac{dC_L}{d\alpha}$	2.093	2.144
$\frac{C_{D_i}}{C_L^2}$	0.1616	0.1600
Distance of bending centre from centre line	0.432s	0.434s
Distance of aerodynamic centre behind mean quarter-chord point	0.0865	0.0405

Figs. 4 and 5 show the comparison of the spanwise lift distribution and the local lift coefficient respectively, while Fig. 6 gives the position of the local aerodynamic centre relative to the quarter-chord line, plotted against  $\eta$ , for the nine-point method only, since the local aerodynamic centre in the simplified method is on the quarter-chord line for each value of  $\eta$ .

It is seen that, except for the aerodynamic centre position, the agreement between the characteristics obtained by the two methods is fairly good. The forward shift of the aerodynamic centre obtained by the second method is due mainly to the neglect of the shift of the local aerodynamic centre from the quarter-chord line.

#### 5 Application to the Effect of Compressibility on Aerofoil II

It is known<sup>6,8</sup> from the linear perturbation theory that the effect of compressibility on the lift distribution of an aerofoil, say Aerofoil II, can be obtained by calculating the lift distribution in incompressible flow of an equivalent aerofoil, say aerofoil I, the lateral dimensions of which have been reduced in the ratio  $\sqrt{1-M^2} : 1$ . The aspect ratio is thus reduced in this ratio, and the sweepback is increased so that

$$\tan \Gamma_1 = \frac{\tan \Gamma_2}{\sqrt{1-M^2}}, \quad \dots\dots\dots (26)$$

where  $\Gamma_1$  is the sweepback of aerofoil I and  $\Gamma_2$  is the sweepback of aerofoil II.

The aerodynamic characteristics of aerofoil II in compressible flow are then obtained from the aerodynamic characteristics of aerofoil I in incompressible flow by multiplication by the following factors:-

Aerodynamic characteristic	Factor
$\frac{dC_L}{d\alpha}$	$\frac{1}{\sqrt{1-M^2}}$
$\frac{C_{D1}}{C_L^2}$	$\sqrt{1-M^2}$
Distance of bending centre on half wing from the centre line, in terms of semi-span	1
$\frac{C_{LL}}{C_L}$ at any $\eta$	1
Distance of aerodynamic centre behind mean quarter chord point, in terms of $\frac{c}{6}$	1

The co-ordinate  $\eta$  for aerofoil II is defined as the distance of a point of the aerofoil from the centre line, divided by the semi-span,  $s'$ , of aerofoil II.

The relations given in the above table have been worked out in Ref.8 by a method similar to that of Ref.7.

The aerodynamic characteristics of Aerofoil I (Fig.1) in incompressible flow, calculated in para.4 above, are used for estimating the aerodynamic characteristics of an Aerofoil II, which has an aspect ratio 4.64, sweep-back  $36^\circ.5$ , taper ratio 3.25:1 (Fig.3), in compressible flow at a Mach number 0.9. The results are given in the following table, which also includes the values of the characteristics of Aerofoil II in incompressible flow, calculated by the simplified method. Comparison of the last two columns therefore shows the effect of compressibility on the aerodynamic characteristics of Aerofoil II.

Aerodynamic characteristic of Aerofoil II	Compressible ( $M = 0.9$ )		Incompressible
	9 point method	Simplified method	Simplified method
$\frac{dC_L}{d\alpha}$	4.86	4.97	3.63
$\frac{C_{D1}}{C_L^2}$	0.0696	0.0690	0.0687
Distance of bending centre from the centre line	0.432s'	0.434s'	0.430s'
Distance of aerodynamic centre behind the mean quarter chord point	0.085 $\bar{c}$	0.040 $\bar{c}$	0.032 $\bar{c}$



Figs. 4-6 still hold for Aerofoil II at  $M = 0.9$  as well as for Aerofoil I in incompressible flow. In Fig. 4, however,  $s'$  must be substituted for  $s$ . Fig. 7 shows the spanwise lift distribution and local lift coefficient calculated by the simplified method for the compressible and incompressible flow round Aerofoil II at the same  $C_L$ , and we see that even at  $M = 0.9$  the effect of compressibility on the distribution is small.

A rough approximation to  $\frac{dC_L}{d\alpha}$  for Aerofoil II in compressible flow can be obtained from the formula for the effect of aspect ratio on a straight wing, based on the simple lifting line theory,

$$\text{i.e.} \quad \frac{dC_L}{d\alpha} = \frac{a_{\infty}}{1 + \frac{a_{\infty}}{\pi A}} \quad \dots\dots\dots(29)$$

In this case we put  $a_{\infty} = (a_{\infty})_0$ , where  $(a_{\infty})_0$  is the lift curve slope of an infinite wing yawed through an angle  $\Gamma_2$  at a Mach number  $M$ , so that

$$(a_{\infty})_0 = \frac{(a_{\infty})_1 \cos \Gamma_2}{\sqrt{1-M^2 \cos^2 \Gamma_2}} \quad \dots\dots\dots(30)$$

with  $(a_{\infty})_1 = 2\pi$ .

This gives a lift curve slope of 4.88 for Aerofoil II at  $M = 0.9$ , and the corresponding formula for Aerofoil II in incompressible flow gives 3.75. These figures agree quite well with the results in the table above. The formula needs further confirmation, both theoretically and experimentally, before it can be accepted in general.

## 6 Conclusions

The above results show that, although the 3-point simplified method gives fairly good agreement with the 9 point method for lift coefficient, local lift coefficient, induced drag, and bending centre, the aerodynamic centre calculated by the 3 point method is approximately 0.058 in front of that calculated by the 9 point method for both aerofoil I, with sweepback  $59^\circ.8$ , aspect ratio 2, in incompressible flow, and aerofoil II, with sweepback  $36^\circ.5$ , aspect ratio 4.64, in compressible flow at a Mach number of 0.9. This difference is sufficient to show that the 3 point method as it stands cannot be used for calculating the aerodynamic centre of an aerofoil with sweepback, but it may be used with good accuracy to determine the other characteristics named above.

As the work involved in the simplified method is very much less than in the full 9-point method, this approximation is of considerable value when a quick estimate of aerodynamic characteristics (except aerodynamic centre) is required. Some work has been done on a second approximation to the simplified method in an attempt to develop a method of calculating the position of the aerodynamic centre much more quickly than by the 9-point method.

LIST OF SYMBOLS

$a_0, a_1, a_2, a_0'$	coefficients in formulae (1) and (19) for the vorticity.
$a_{\infty}$	lift curve slope in two-dimensional flow.
$(a_{\infty})_0$	two-dimensional lift curve slope of an infinite wing yawed through an angle $\frac{1}{2}$ , at a Mach number $M$ .
$(a_{\infty})_i$	two-dimensional lift curve slope of an unyawed infinite wing in incompressible flow.
$A$	aspect ratio
$A_1, A_3, A_5, A_1', A_3', A_5'$	coefficients defined in equations (6), (7), (8), (23), (24), (25).
$c$	chord of the aerofoil.
$\bar{c}$	mean chord of the aerofoil.
$c_0, c_1, c_2, c_0'$	coefficients in formulae (1) and (19) for the vorticity.
$C_L, C_{D1}$	total lift coefficient and induced drag coefficient.
$C_{LL}$	local lift coefficient, defined as $C_{LL} = \frac{dL}{dy} / \frac{1}{2} \rho V^2 c$
$e_0, e_1, e_2, e_0'$	coefficients in formulae (1) and (19) for the vorticity.
$F_0(\eta), F_1(\eta), F_2(\eta)$	function of $\eta$ used in equation (2).
$G_0(\eta)$	a function of $\eta$ used in equation (19).
$k$	vorticity, a function of $\eta$ and $\theta$ .
$k_1, k_2, k_3, k_4$	strength of the four chordwise vortices at any section spanwise.
$K$	circulation round a section of the aerofoil (a function of $\eta$ ).
$K_1, K_2, K_3$	defined in formula (18) for $k$ .
$L$	lift on the aerofoil.
$M$	Mach number.
$s, s'$	semi-spans of Aerofoils I and II respectively.
$S$	area of aerofoil I.
$V$	velocity of flow at infinity.
$x$	distance of any point of the aerofoil in front of the mid-point of the chord through it.

LIST OF SYMBOLS (Contd.)

$x$	distance of the local aerodynamic centre at any spanwise position behind the local quarter chord point, in terms of local chord.
$y$	distance from the centre line to any point of the aerofoil (positive to starboard).
$\alpha$	angle of incidence from zero lift.
$\Gamma_1, \Gamma_2$	angles of sweepback of aerofoils I and II respectively.
$\phi$	$\cos^{-1} \left( \frac{-y}{s} \right)$
$\eta$	$y/s$
$\theta$	$\cos^{-1} \left( \frac{2x}{c} \right)$

LIST OF REFERENCES

<u>No.</u>	<u>Author</u>	<u>Title, etc.</u>
1	V.M. Falkner	The Calculation of Aerodynamic Loading on Surfaces of any Shape. A.R.C. Report 6997. Aug., 1943.
2	V.M. Falkner	The Calculation of Aerodynamic Loading on a Swept-back Wing. A.R.C. Report 7322. Jan., 1944.
3	Mutterperl	The Calculation of Span Load Distribution on Swept-back Wings. N.A.C.A. Tech. Note 834. Dec., 1941.
4	Weissinger	"Über die Auftriebsverteilung von Pfeilflügeln." F.B. 1553. 1942.
5	V.M. Falkner	The Simplification of Wing Loading Calculations by Lifting Plane Theory. A.R.C. Report 9211. Dec., 1945.
6	B. Göthert	Berechnung des Geschwindigkeitsfeldes von Pfeilflügeln bei hohen Unterschallgeschwindigkeiten. Lilienthal-Gesellschaft für Luftfahrtforschung. Bericht 127, p. 32. Sept. 1940.
7	Goldstein Young	The linear Perturbation Theory of Compressible Flow, with applications to Wind-Tunnel Interference. A.R.C. Report 6865. July, 1943.
8	R. Dickson	The Comparison between the Compressible Flow round a Swept-back Aerofoil and the Incompressible Flow round Equivalent Aerofoils (in preparation).

Attached:

Tables I and II  
Drg. Nos. 18865.S - 18871.S (7)

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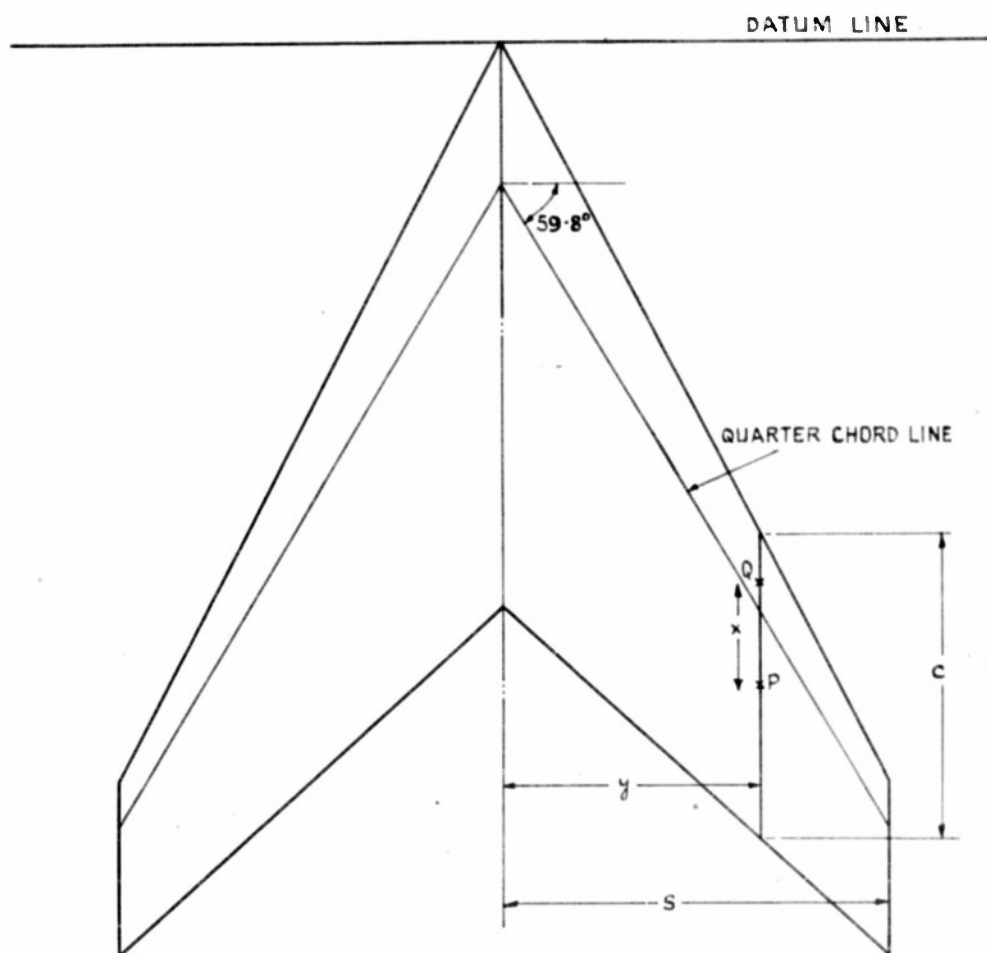
TABLE ICoefficients in the 9-point solution

$a_0 = 0.06699$	$c_0 = 0.07122$	$e_0 = 0.07865$
$a_1 = 0.05901$	$c_1 = -0.14683$	$e_1 = -0.07556$
$a_2 = 0.00799$	$c_2 = 0.03235$	$e_2 = -0.09512$

TABLE IICoefficients in the  
Simplified (3-point) solution

$a_0' = 0.104929$
$c_0' = 0.008992$
$e_0' = 0.011573$

FIG. 1.



TAPER RATIO = 3.25 : 1.

ASPECT RATIO = 2

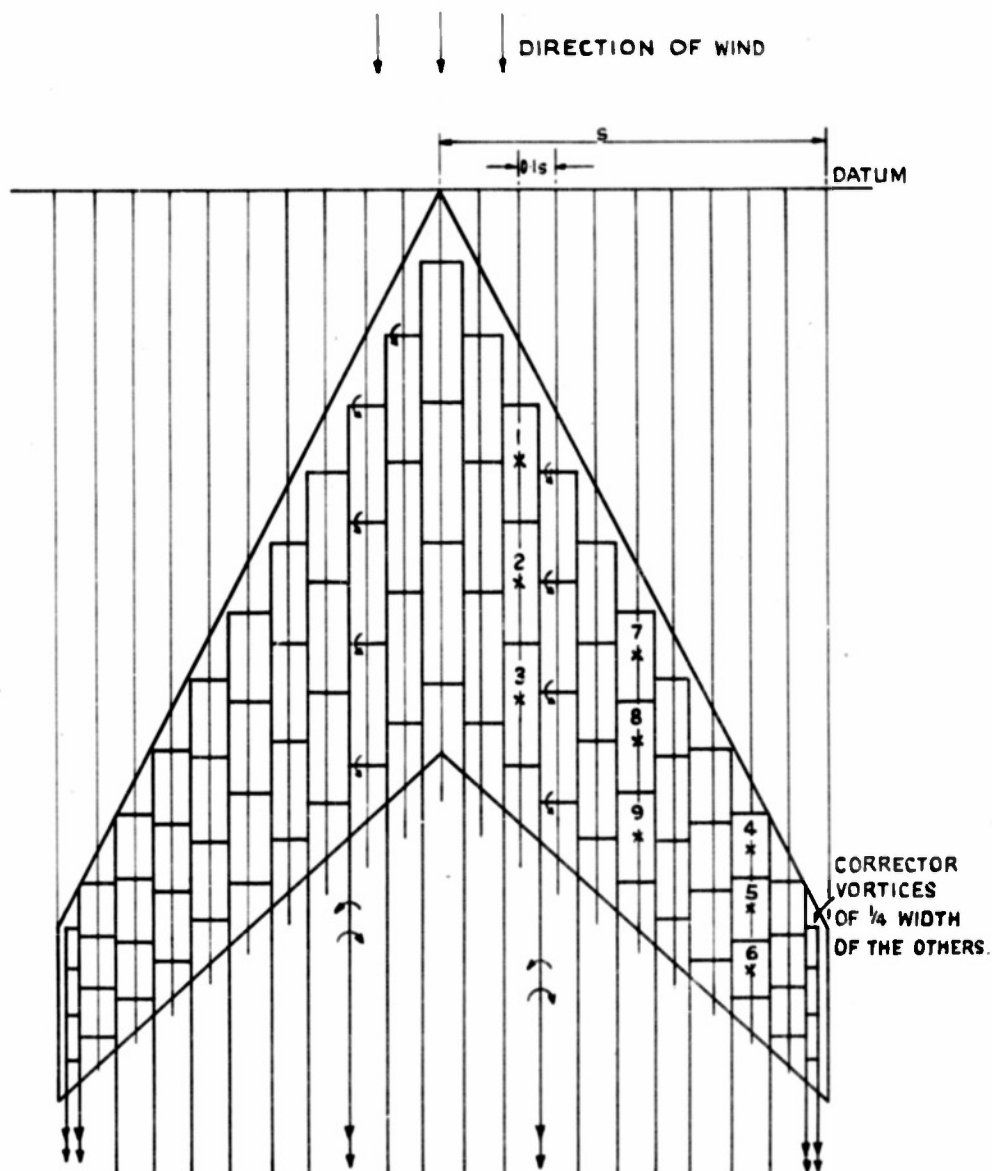
AT A GENERAL POSITION SPANWISE,  $y = S \eta = -S \cos \phi$ .

ON THIS CHORD  $C$ ,  $P$  IS THE MID POINT,  $Q$  IS AN ARBITRARY POINT.

$$PQ = x = \frac{C}{2} \cos \theta.$$

AEROFOIL I.

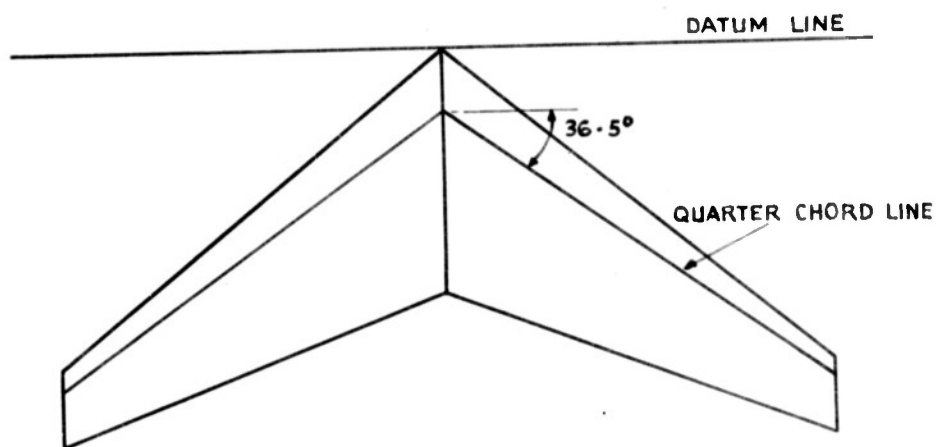
FIG. 2.



PIVOTAL POINTS ARE SHOWN BY CROSSES.

PATTERN OF HORSESHOE VORTICES  
REPRESENTING CONTINUOUS LEADING.

FIG. 3.



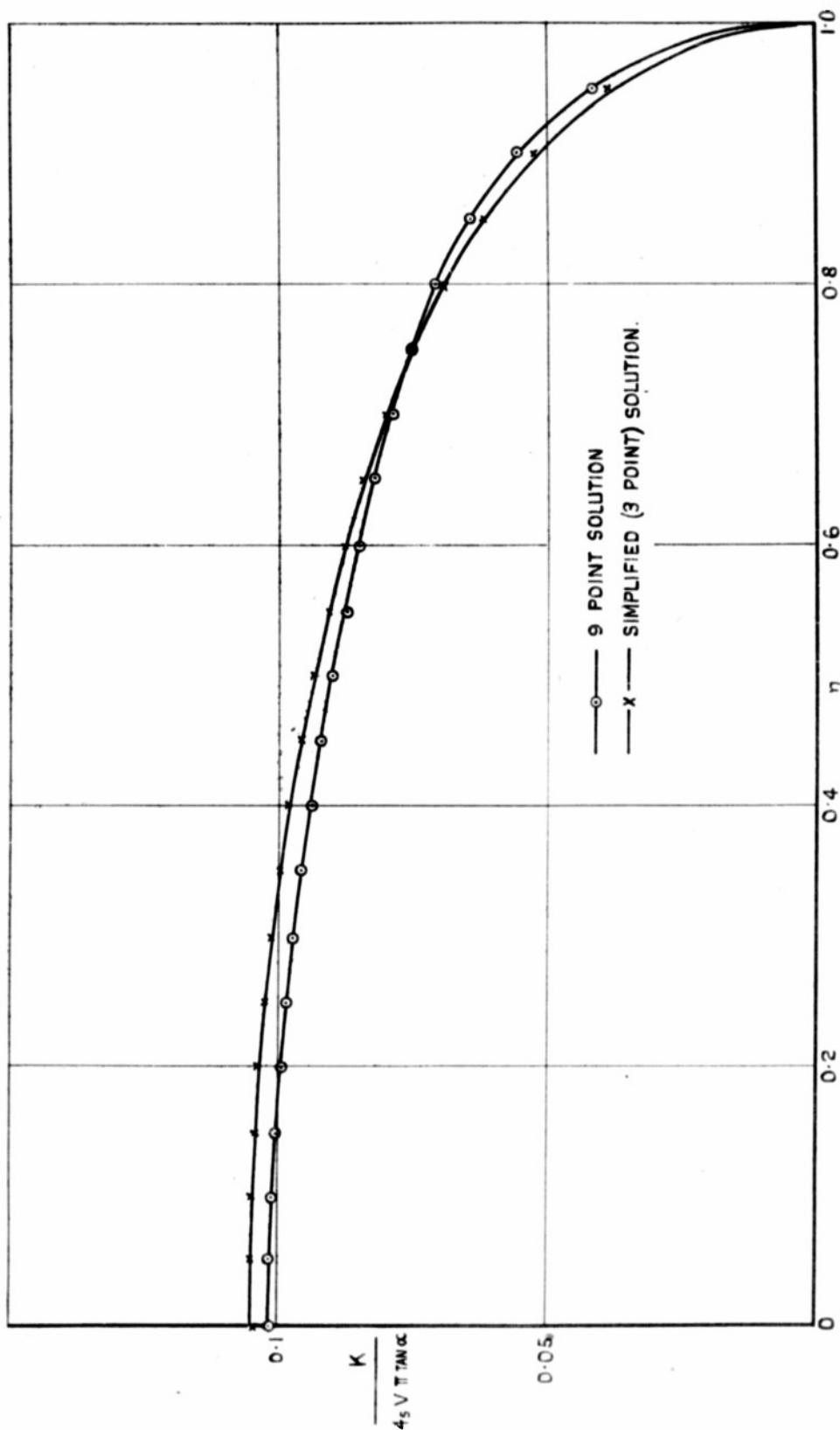
TAPER RATIO = 3.25 : 1

ASPECT RATIO = 4.64.

AEROFOIL II.

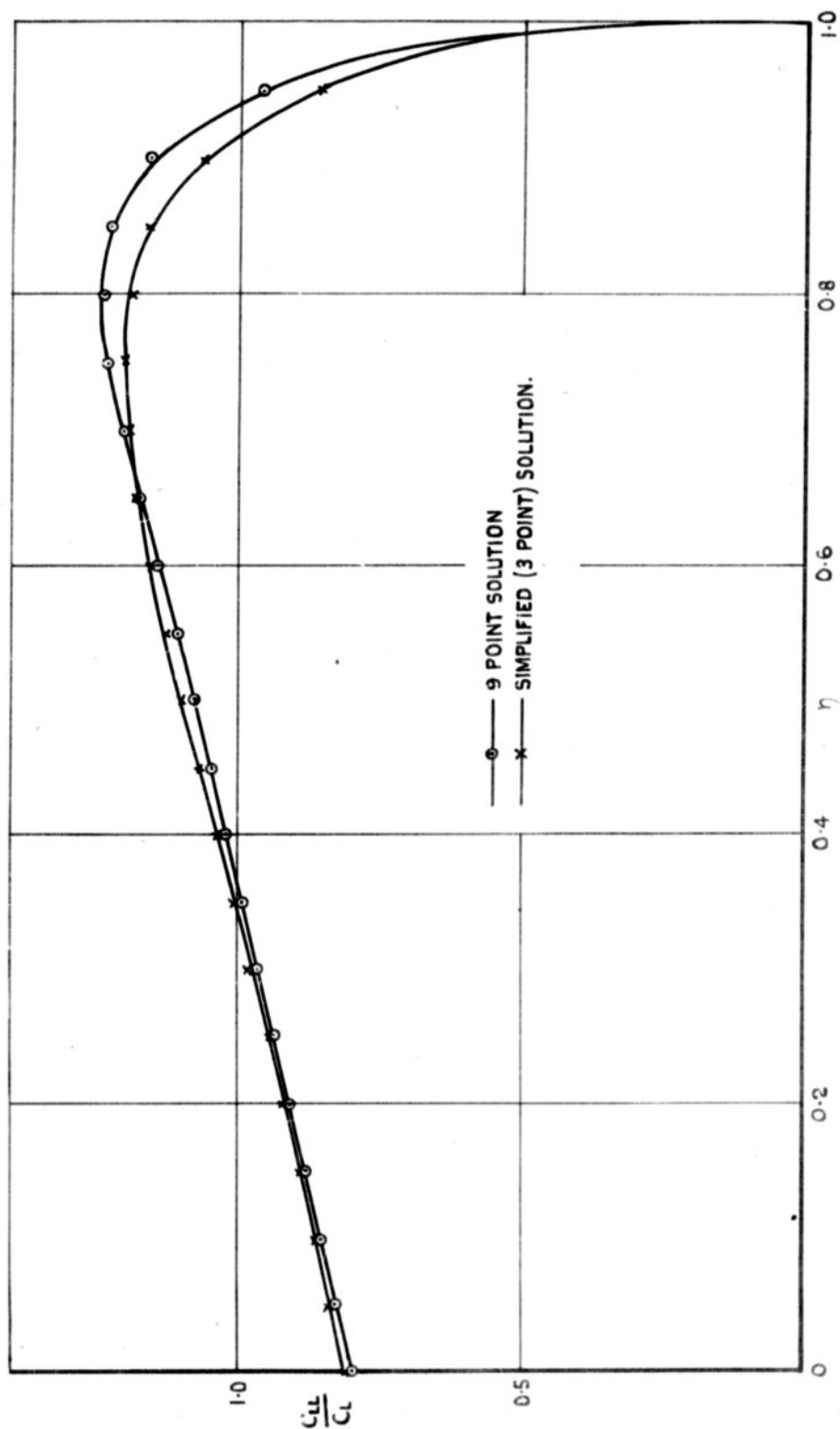


FIG. 4.



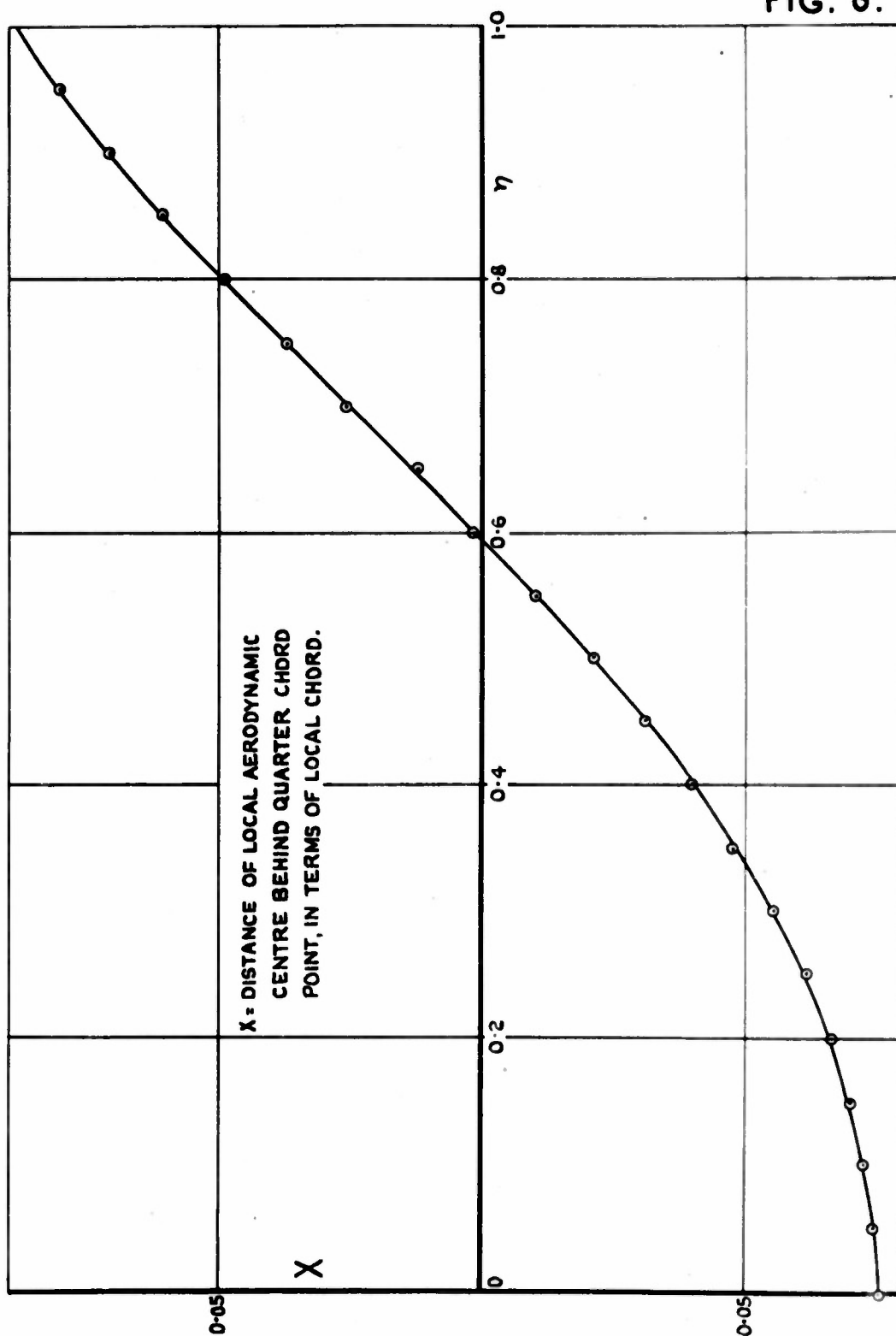
DISTRIBUTION OF CIRCULATION ON AEROFOIL I  
(PROPORTIONAL TO SPANWISE LIFT DISTRIBUTION).

FIG. 5.



LOCAL LIFT COEFFICIENT FOR AEROFOIL 1.

FIG. 6.



POSITION OF LOCAL AERODYNAMIC CENTRE RELATIVE TO THE QUARTER CHORD LINE.

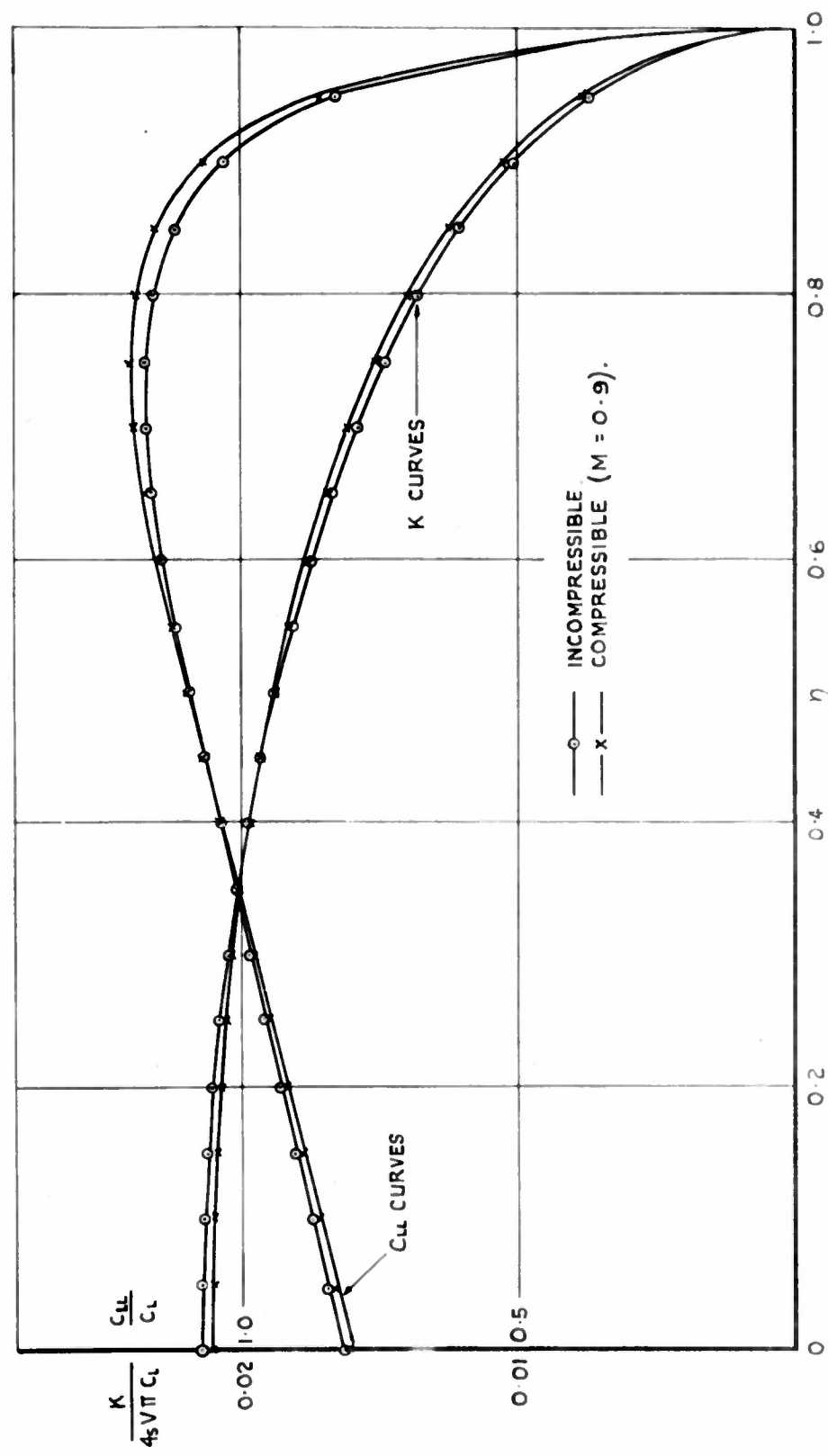


FIG. 7.

LIFT DISTRIBUTION AND LOCAL LIFT COEFFICIENT FOR AEROFOIL II IN COMPRESSIBLE AND INCOMPRESSIBLE FLOW, USING SIMPLIFIED METHOD.

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REPORT NO. A (R) 100

Dickson, R.

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DIVISION: Aerodynamics (2)

SECTION: Wings and Airfoils (6)

CROSS REFERENCES: Airfoils - Aerodynamics (07710);  
Airfoils - Swept-back - Drag (08260)

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AUTHOR(S)

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ORIGINATING AGENCY: Royal Aircraft Establishment, Farnborough, Hants

TRANSLATION:

COUNTRY	LANGUAGE	FORG'N. CLASS.	U. S. CLASS.	DATE	PAGES	ILLUS.	FEATURES
Gt. Brit.	Eng.	Rostr.	Rostr.	Jun '46	23		tables, graphs, drwgs

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